PRACTICE MIDTERM 1 (CHRIST) - BRIEF SOLUTIONS

PEYAM RYAN TABRIZIAN

(1a) $\boxed{\frac{1}{6}}$ (use the conjugate form, or the fact that $x - 9 = \left(\sqrt{x}\right)^2 - 3^2$ since $x > 0$)

(1b) $\boxed{f'(9) = \frac{1}{6}}$, you've calculated that on question **(1a)**

(2a) Horizontal asymptote: $\boxed{y = \frac{1}{3}}$ (at $\pm\infty$)
Vertical Asymptotes: $\boxed{x = 1 \text{ and } x = 3}$ (Note that there is no vertical asymptote at $x = 8$ since the limit at 8 exists)

(2b) $\boxed{0}$ (by the squeeze theorem)

(3) By the Intermediate Value Theorem (let $f(x) = x^6 - 1 - \sin(x)$, and choose for example $a = 0$ and $b = \pi$ and say that f is continuous)

(4a) $\log_r(2)$ is the unique number a such that $r^a = 2$

(4b) $\boxed{\text{Yes}}$ (by the horizontal line test); Domain = Range of f = $\boxed{(-\infty, 0)}$; Range = Domain of f = $\boxed{\left(-\frac{\pi}{2}, 0\right)}$

(4c) It's not the graph of a function (by the vertical line test)

(4d) $\boxed{\ln(5) + 1 + \frac{1}{2}\ln(x)}$

(4e) $\boxed{\text{No! Let } f(x) = |x|}$

(5a) $\delta = \min\left\{1, \frac{\epsilon}{13}\right\} = \min\left\{1, \frac{\frac{1}{1000}}{13}\right\} = \min\left\{1, \frac{1}{13000}\right\} = \boxed{\frac{1}{13000}}$ (it's just like the complicated case of $\epsilon - \delta$, except that $\epsilon = \frac{1}{1000}$

(5b) This is **HARD**, you can e-mail me about that, or ask me about that during office hours if you want, there will be nothing like that on the exam! In case you're curious, $\boxed{\delta = \min\left\{\frac{1}{6}, \frac{\epsilon}{45}\right\}}$

Date: Monday, February 18th, 2011.